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CRITICAL FLOW METERS (VENTURI FLUMES)

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HYDRAULICS DIVISION

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CRITICAL FLOW METERS
(VENTURI FLUMES)

Armando Balloffet,[1] A.M. ASCE

SYNOPSIS

Results of experiments on critical flow meters made at the Hydraulics Laboratory of Buenos Aires University, and their applications to open channel discharge measurements are reported in this paper. Standardization, as a means for saving costs and facilitating field operations, has been the primary purpose of these studies.

INTRODUCTION

An important problem in hydraulic engineering is the measurement of discharge in open channels.

A good national policy controlling the logical use of available water, particularly in irrigation systems, demands a simple, practical way of accurately measuring the water flow. Not only must the water be measured in the main supply channels, but also in the last farm ditch.

The problem is to develop a measuring device which holds its accuracy under adverse conditions, has no moving parts, neither causes nor is affected by deposition of sediment, and is inexpensive. Since this type of meter is likely to be used by inexperienced help, the readings must be readily evaluated into discharge.

Venturi Flumes or Critical Flow Meters

The above conditions can be fulfilled in most cases by a Venturi flume. Its basic principle consists in the formation of critical flow in a section of the channel, which may be produced by installing a contraction. If the meter is to be placed in an existing canal, the necessary upstream head is determined by means of a backwater curve.

With critical flow at the "throat," the discharge varies only with the upstream depth, independently of the state of flow downstream. If this downstream flow is tranquil, a hydraulic jump takes place.

This is the reason for the term: "hydraulic jump meters," often employed in Spanish, French or Italian literature, while English writers use "Venturi meters or flumes." This term is probably originated from early study of true Venturi canals.

It is believed that both designations are misleading because neither the Venturi nor the hydraulic jump principles are applied in the discharge formula. Thus, the designation: "critical flow meters," as suggested by the late Professor Bakhmeteff,[(2)] seems preferable.

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1. Associate Professor of Hydraulics, University of Buenos Aires, Argentina. With the collaboration of Messrs. O. Lehmann and A. Velasco, Civ. Engrs.
2. Bakhmeteff: "Hydraulics of Open Channels," McGraw-Hill Book Co., New York, 1932, p. 44.

Theoretical Analysis

The discharge equation for critical flow meters can be derived from Bernoulli's equation. On this theoretical basis, this discharge may be computed with an error not exceeding 4 or 5 per cent, either for the models discussed herein or for those studied by Professors De Marchi and Contessini in Milan.[3]

From the Bernoulli formula for steady flow of perfect liquids:

$$H = h + V^2/2g = \text{const.} \qquad (1)$$

In which:

H = Total head;
h = Piezometric head;
V = Mean velocity.

It is assumed that the Coriolis' coefficient ($\alpha = \int_A v^3 dA/V^3 A$, in which A is the total area of flow and v the individual velocity of each point of cross section) is practically unity.

For non-curvilinear flow in an open channel with horizontal bottom, equation (1) can be referred to the bottom as a datum plane. In this case, h equals the depth in each section, and the total head is said to be the specific head.

If the section of channel is rectangular, V = Q/by, in which Q is the discharge; b, the width and y = h is the depth. In this case, the critical depth is:

$$y_c = \sqrt[3]{Q^2/gb^2} \; ; \qquad (2)$$

and

$$H_{min} = 3/2 \; y_c \; , \qquad (3)$$

in which H_{min} is the minimum specific energy corresponding to the discharge Q.

For the condition of tranquil flow upstream from the meter, the specific energy must exceed the minimum. For the same discharge, the narrow throat will induce critical depth, and if no losses are considered, the same specific energy will be a minimum in this section. Thus:

$$H = y + V^2/2g = 3/2 \; y_c \; ; \qquad (4)$$

and

$$y + Q^2/2gB^2y^2 = 3/2 \; y_c \; ; \qquad (5)$$

in which B is width and y is depth of upstream canal and y_c is critical depth at the throat.

Throat sill.—For meters with a throat sill, the elevation of the sill is used as a datum. Combining (5) and (2), there is obtained:

$$y + \frac{y_c^3 b^2}{2B^2y^2} = 3/2 \; y_c \; , \qquad (6)$$

in which b is now the width of throat section.

If, according with Jameson[4] and De Marchi and Contessini (op. cit.), there

3. De Marchi and Contessini: "Dispositivi per la misura della portata dei canali con minima perdita di quota." L'Energia Elettrica, jan. 1936-march 1937.
4. Jameson: "The diversion of the river Ashop," Discussion of a paper by Scott Thompson. Proc. Inst. Civil Engineers. London. p. 411, vol. 229, 1929/30.

is substituted:

$$M = (B/b)^2 = 1/r^2 \quad \text{and} \quad z = y_c/y \ ,$$

in which r is the contraction ratio, there results:

$$z^3 - 3Mz + 2M = 0. \tag{7}$$

This equation has three real roots. One is negative, and must be disregarded. From the other two positive roots the least must be selected, which corresponds to tranquil upstream flow. Then:

$$z_1 = 2\sqrt{M} \cos\left(\frac{\pi + \cos^{-1}\sqrt{1/M}}{3}\right) . \tag{8}$$

Whence:

$$z_1 = \frac{2}{r} \cos\left(\frac{\pi + \cos^{-1} r}{3}\right) . \tag{9}$$

But from (2):

$$y_c^3 = Q^2/gb^2 = z_1^3 \, y^3 \ ;$$

and then:

$$Q = z_1^{3/2} \, by\sqrt{gy} \ ;$$

and finally:

$$Q = \mu \, by\sqrt{2gy} \ . \tag{10}$$

In which:

$$\mu = \frac{z_1^{3/2}}{\sqrt{2}} = 2\sqrt{1/r^3 \cdot \cos^3(\pi/3 + 1/3 \cos^{-1} r)}. \tag{11}$$

Here, μ depends on r, is constant for each meter, and is independent of the head, when the bottom is straight.

Equation (10) is similar to the weir formula, except that μ varies with the contraction ratio in the following range:

when r approaches 0, μ tends to become .385;
when r approaches 1, μ tends to become .707.

The lower value of μ is equal to the theoretical discharge coefficient for a broad-crested weir, when its upstream velocity head is negligible. Inasmuch as the flow over a weir passes from tranquil, through critical, to shooting flow, the weir may be considered as a critical flow meter.

The upper value of μ corresponds to a free overfall with no lateral contraction.

Coefficient C

Under the present assumptions, h is measured by y and there are no losses. However, since the flow is not completely straight and some losses occur, a coefficient C must be introduced into equation (10):

$$Q = C\mu \, by\sqrt{2gy} \ . \tag{12}$$

Curvilinear flow leads to greater values of C, as found by Khafagi,[5] probably due to its importance in the very short throat sections of his models.

5. Khafagi: "Der Venturikanal," Technische Hochschüle, Zurich, 1942.

Submergence

The minimum upstream depth for a given discharge occurs when flow in the throat is critical. Equation (12) is applicable only in this condition. When there is tranquil flow in this section, variations in the downstream depth influence upstream depth, and the meter is said to be submerged.

Prof. L. Escande[6] has studied the flow between bridge piers which in some cases corresponds closely to the flow through a meter. This author describes three classes of flow. First, there is free flow, in which the liquid issues from the throat as shooting flow. The hydraulic jump, if any, is well downstream. In the second case, there is a jet flowing under the tailwater with a surface roller with velocity opposite to that of the jet. In the third case, when the tailwater rises, the jet changes completely, and main flow reaches the free surface. This type of flow has been named by Escande as axial flow (écoulement axial) and is wholly tranquil.

Previous experiments[7] have shown that only when the jet flows free, is the upstream depth unaffected by variations in the tailwater depth. When the jet is submerged there is a small influence, but equation (12) could still be used within certain limits of error. Values of C tend to lower as downstream depth increases.

Flow with submerged jet seemed to be a transition between free and completely submerged flow.

For practical use of the meter it is very important to designate the type of flow by an index number, which is the ratio between downstream and upstream depths above throat bottom. This index has been called the degree of submergence. When it exceeds a certain limit, the meter is said to be submerged and and one depth influences the other.

According to De Marchi and Contessini (op. cit.), the point of submergence can be theoretically determined. As indicated in a previous paper,[8] two assumptions are made:

a) The hydraulic jump extends across the canal from one wall to the other;
b) There is no head loss up to the jump.

De Marchi and Contessini's third assumption that the jump is a sudden rise in water level is not necessary here, since the bottom is level.

In Fig. 1 main conditions of flow through the meter are shown. The distance between the total upstream head line and the free surface downstream from the jump is called Δ. Point of submergence occurs when, with critical flow in throat, Δ is minimum.

If y' is the depth just upstream from the jump, then:

$$H = 3/2\ y_c = y' + \frac{Q^2}{2\,g y'^2 B^2} \quad , \tag{13}$$

in which B is the downstream width of canal.

Equation (13) may be written also:

$$H = 3/2\ y_c = y' + y_c'^3 / 2y'^2 \quad , \tag{14}$$

6. Escande: "Étude de quelques écoulements comportant la formation d'une veine de coutant." Publ. Scient. et Techniques du Ministère de l'Air. Paris. 1946.
7. Balloffet: "Estudios sobre aforadores de resalto." Report to the Engineering Convention, Rio de Janeiro, July, 1949.
8. Balloffet: "Aforadores de resalto," Ciencia y Técnica, Buenos Aires, Vol. 112, No. 559. Jan. 1949.

in which y'_c is the critical depth for discharge Q and canal width B, which are related to y_c as follows:

$$y_c = \sqrt[3]{Q^2/gb^2} \quad \text{and} \quad y'_c = \sqrt[3]{Q^2/gB^2} \ , \tag{15}$$

from which:

$$y_c^3 b^2 = y_c'^3 B^2 \quad \text{and} \quad y_c'^3 = r^2 y_c^3 \ . \tag{16}$$

Consequently:

$$3/2 y_c = y' + r^2 y_c^3 / 2y'^2 \ . \tag{17}$$

However, downstream from the jump the depth y_a has the following relationship to y':

$$y_a = -y'/2 + \sqrt{y'^2/4 + 2y_c'^3/y'} \tag{18}$$

and combining (16) and (18), and substracting from (17), there is obtained:

$$\Delta = 3/2\, y_c - y_a = 3/2\, y' + r^2 y_c^3 / 2y'^2 - \sqrt{y'^2/4 + 2r^2 y_c^3/y'} . \tag{19}$$

Now, according to Bakhmeteff, $\lambda = y_c'^3/y'^3$ and:

$$\Delta / y_c = \frac{1}{2\sqrt[3]{\lambda}} \left(3 + r^2\lambda - \sqrt{1 + 8r^2\lambda}\right) .$$

Differentiating and equating to zero:

$$8r^6\lambda^3 - 27r^4\lambda^2 + 17r^2\lambda + 2 = 0 \ . \tag{20}$$

Dividing (20) by $r^2\lambda - 1$,

$$8r^4\lambda^2 - 19r^2\lambda - 2 = 0 \ .$$

which is an equation of the second degree in $r^2\lambda$. The positive root is:

$$r^2\lambda_1 = 2.4760 \ .$$

Then, the minimum value of Δ/y_c is:

$$\Delta_{min} / y_c = 0.33787\ r^{2/3} \ ;$$

and

$$y_{a1} = y_c\,(3/2 - 0.33787\ r^{2/3}) \ , \tag{21}$$

in which y_{a1} is the value of the maximum downstream depth required to maintain critical flow in the throat, for the given discharge.

Now, assuming no head losses,

$$Q = \mu by\sqrt{2gy} \ ,$$

and therefore:

$$Q^2/gb^2 = 2\mu^2 y^3 = y_c^3 \ , \tag{22}$$

and, consequently:

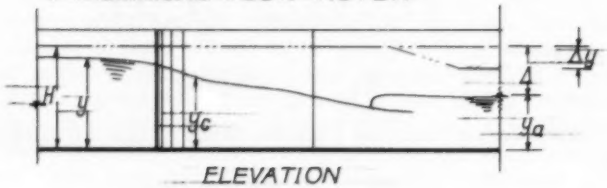
$$y = \frac{1}{\sqrt[3]{2\mu^2}}\ y_c \ . \tag{23}$$

Finally, the limit of submergence, S_1, i.e. the ratio between (21) and (23), may be written in the following manner:

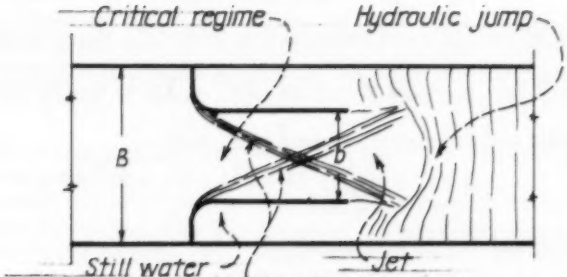
$$S_1 = \sqrt[3]{2\mu^2}\ \ (3/2 - 0.33787\ r^{2/3}) \ . \tag{24}$$

Fig. 1

CRITICAL FLOW METER



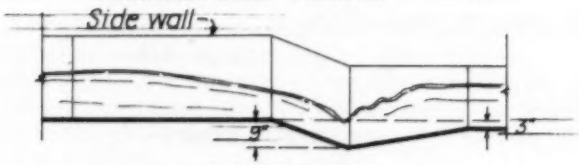
ELEVATION



PLAN

Fig. 2

PARSHALL FLUME



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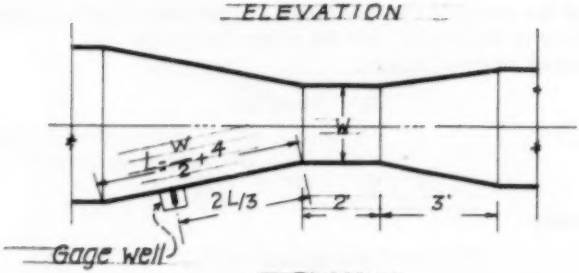


PLAN

Fig 3

CRUMP MODEL

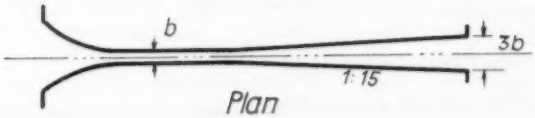


Fig 4

INGLIS MODEL

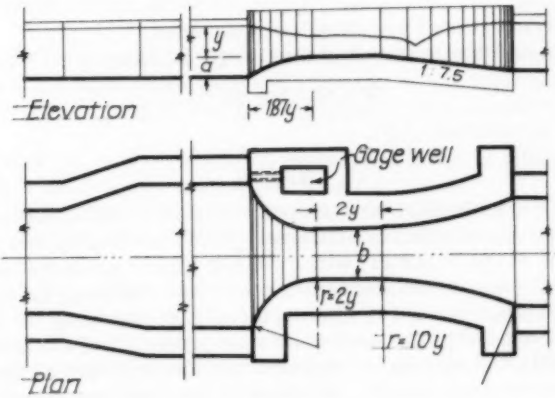
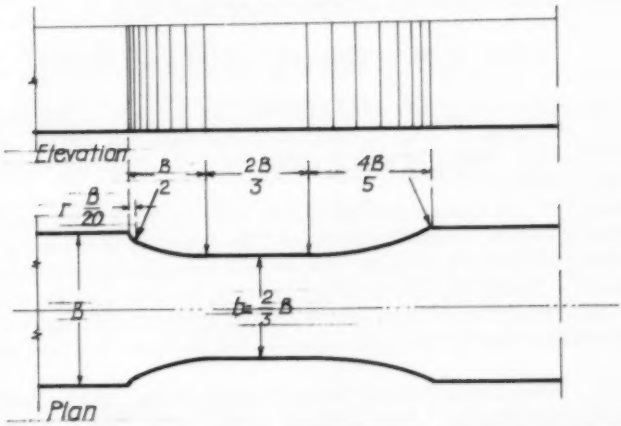


Fig 5

DE MARCHI MODEL



This relationship gives $S_1 = .91$, for $r = 1/3$, and $S_1 = .90$, for $r = 2/3$

If it is assumed that the coefficient of discharge, C, is .95 and .96, respectively, for meters with $r = 1/3$ and $r = 2/3$, $S_1 = .88$ for both cases.

These expressions are subject to some qualifying factors. The width of the hydraulic jump is not quite equal to the width of canal, but has a somewhat curvilinear form. Also, paths of the jet are not perfectly straight, as assumed. Finally, as in the "PD" type of meters tested, the width of critical section could be indeterminate.

Significance of Discharge Coefficient

The degree of precision with which results of tests could be applied to large scale field conditions was an important question in these studies.

If flows through two similar meters are dynamically similar, the corresponding Froude numbers must be equal:

$$F = V_1 / \sqrt{gy_1} = V_2 / \sqrt{gy_2} \ , \qquad (25)$$

assuming that gravitational acceleration is constant.

But F may be written in terms of discharge, as follows:

$$F = Q / By\sqrt{gy} \ . \qquad (26)$$

Yet, according to equation (12):

$$F = \sqrt{2}\, C\mu r \ . \qquad (27)$$

If two meters are similar, both μ coefficients are equal, since they depend only on r. However, discharge coefficient C is influenced by two principal factors, namely the shape of meter, which causes form resistance and its surface roughness, which causes surface resistance. Other factors, as surface tension or air resistance can usually be neglected. The first factor is constant if both meters are similar. The second (surface resistance) could vary if the materials of walls and bottom, or dimensions of meters are different. However, in the present experiments, influence of relative roughness—the relation between absolute mean roughness and some reference length—was negligible. Thus, if two meters are geometrically similar, both flows are dynamically similar, at least within the range covered by this experimentation, because Froude numbers are equal to a constant multiplied by C.

Review of Some Types of Critical Flow Meters

One of the most extensively used critical flow meters is the flume developed by R. L. Parshall,[9] Fig. 2. It is a good measuring device, although its empirical equations cannot be safely applied to conditions different from the original experiments.

Elsewhere,[10] the theoretical equations for free flow were applied to a Parshall flume, with a throat width which was different from the original. Tests showed a difference between theoretical and experimental results. It was reasoned[7] that it was impossible to know the exact location of the critical section and, thus, the bottom elevation and width of that section were unknown. This meter has a non-prismatic throat section. Furthermore, it is

9. Parshall: "Measuring Flume," Fort Collins, Colo., 1921. Ibid. "The Improved Venturi Flume," Fort Collins, Colo., 1928; Ibid. "Measuring Water in Irrigation Channels," U. S. Department of Agriculture, 1941.
10. Balloffet, "Un aforador de resalto," Revista ANDA, No. 108, Buenos Aires, June, 1946.

probable that the critical section varies with discharge. Villemonte and Gunaji[11] have studied the submerged condition of a 6-in throat width Parshall flume and found that the point of submergence was .56, and, for bigger values of S, the submerged sharp crested weir formula was applicable.

Fane, in India, some 25 years ago, applied the principle of critical flow to discharge measuring. Crump, also in India,[12] developed a good flow meter. In Fig. 3 can be observed the plan of Crump's horizontal-bottom model. Sir C. Inglis, also in India in 1928 studied a model, Fig. 4, considerably shorter and comparable to those further tested by De Marchi and Contessini. The studies of Inglis were very useful for the observation of the measuring device, operating as a critical flow meter. Studies up to that date referred mainly to the Venturi principle and depths were measured at the entrance and throat sections. The piezometric tap of this model is directed downstream as shown in Fig. 4. This can lead to some error, since it can receive some velocity head.

Other experiments were carried on by Engel, in London, some 25 years ago.

In 1936, the experiments by De Marchi and Contessini, already quoted,[3] were published. It is believed that theirs is one of the most outstanding contributions on this subject. The shape of a typical Milan model is reproduced in Fig. 5.

In 1941, a new paper by Citrini[13] appeared with a basis for standardization of models studied by De Marchi and Contessini.

In 1942, in Switzerland, Khafagi[5] published his thesis on experiments carried on under the direction of Prof. Meyer Peter. These studies were very accurate.

Nearly all of the models mentioned above had rather great lengths, apparently in order to lower the head loss, in much the same way as a Venturi meter in a pipe. However, the greatest part of the loss was due to the hydraulic jump, which is entirely different from the flow in a closed conduit.

Models Tested at the Hydraulic Laboratory of Buenos Aires University

In fact, only in case of supercritical flow of a gas through a divergent section of conduit is it possible to obtain a shock wave in pipe flow. In this case there is analogous to the hydraulic jump. Obviously supersonic flow of liquid in a pipe is practically impossible.

When a liquid flows through a Venturi tube, the turbulence is greater when the boundary has rapid divergence downstream from the throat section. Fluid flow separates[14] with a subsequent increase in head loss. Consequently, to save head loss in a pipe, a long Venturi tube must be used. However, there are instances in which it is quite possible to use less expensive, short Venturi tubes, or nozzles, or even simple pipe orifices.

In an open-channel, critical flow meter, the effect of separation is much smaller. When the bottom is level, it can occur only at the walls. As far as the free surface is concerned, the air resistance is negligible.

Now, if the cross section is rectangular and the walls have little or no divergence, the head loss of jump depends primarily on its upstream and downstream depths.

---

11. "Equation for Submerged Sharp-crested Weirs Found Applicable to 6-in Parshall Flume." Civil Engineering, Vol. 23, No. 6. June 1953.
12. Citrini: "Misuratori a risalto." Politecnico di Milano. 1939.
13. "Modellatori a risalto." Politecnico di Milano. 1941.
14. Rouse: "Engineering Hydraulics," Wiley, N. Y., 1950, p. 115.

Consequently, it may be reasoned that a gradual expansion would be of little or no significance in lowering the head loss.

Models.—The models tested at the Buenos Aires University were made without any gradual expansion at the exit.

Two different patterns were used. The first type, "A" and "B" models (Fig. 6), was made in a rectangular canal by means of one or two vertical sheets. The throat was formed either between both sheets or between a sheet and a free wall, as the case might be.

The second type, Fig. 7, corresponded to "PD" models, and was even simpler. Here the sheets were plane, vertical, and at right angles with the walls. At what corresponded to the throat, they had a sharp edge.

In all cases, the bottom was level. As it was intended to obtain a series of standard models, only fixed proportions and contraction ratios (r) of 1/3 and 2/3 were used.

Experimental Set-up

The hydraulics laboratory of the Buenos Aires University has a long rectangular canal with lucite walls. Bottom slope can be changed by means of a threaded post at the downstream end. However, throughout this series of experiments it was kept horizontal.

Water was pumped from an underground reservoir, passed through the experimental section, and returned through the control meter to the reservoir.

The control meter was a triangular weir located in a stilling basin with the exact dimensions of the one used by De Marchi and Contessini in Milan.

Photograph I is a general view of the apparatus.

The maximum discharge used was 30 liters per second (a little more than 1 cfs).

Initially the width of canal was 38.9 cm (15.4 in). Later, after changing the walls, the width was 40.2 cm (15.8 in).

Tests Performed

The experiments had three different phases.

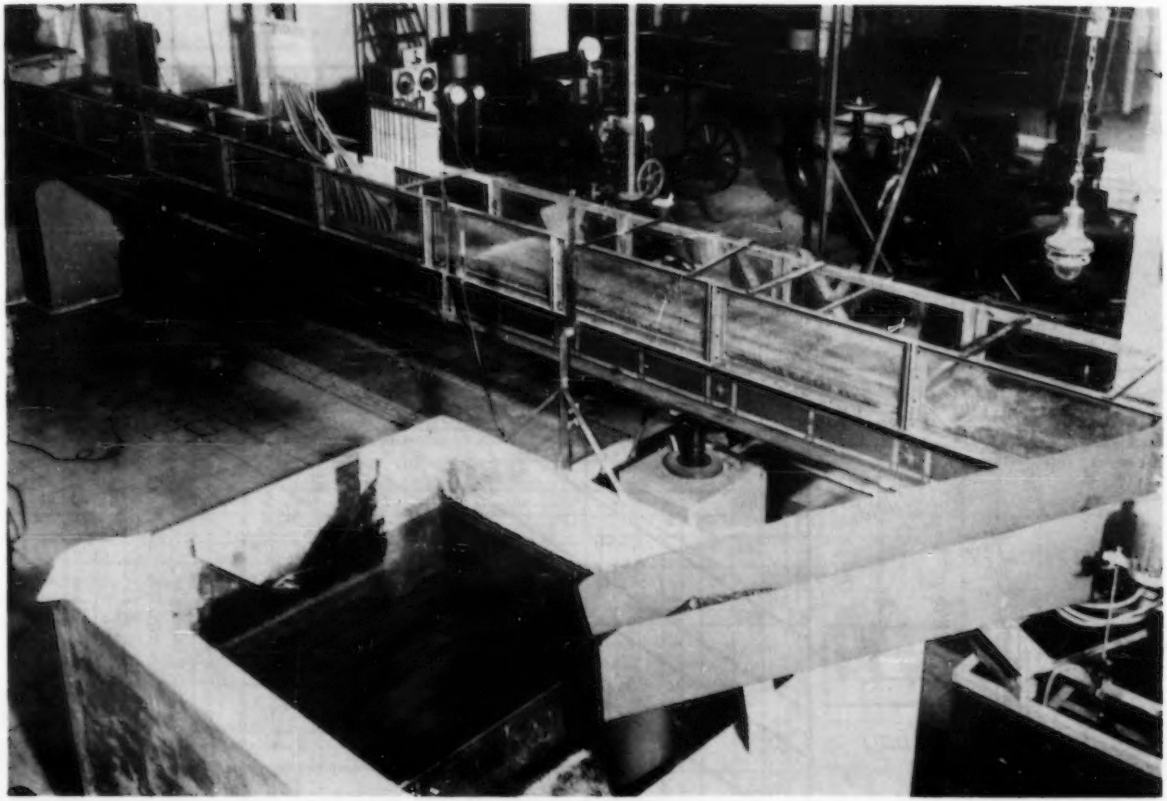
a) First the discharge coefficient (C) was determined. For this purpose, free-flow or shooting flow was established downstream. Then, the head y, (at a section some 4 ft upstream from the entrance of meter) and the downstream head, y_a, were measured, and the discharge was determined by the control weir.

The ratio of the actual flow to the theoretical flow (equation 12) is the discharge coefficient C.

b) The point of submergence was determined. For this purpose a certain discharge was set. Then the downstream level was raised slowly by means of a little hinged leaf gate located at the end of canal. This operation was suspended when the upstream level became affected. This process was repeated for several different discharges.

c) Finally the pressures on the bottom and the corresponding surface profiles were determined by means of a differential manifold manometer and a series of point gages fixed to a frame.

Differences between depths and pressures (expressed in water column units) showed effects of path curvature. The critical depth was calculated and the type of flow in the throat determined.



Photograph I.

General View of the Test Canal and Control Weir

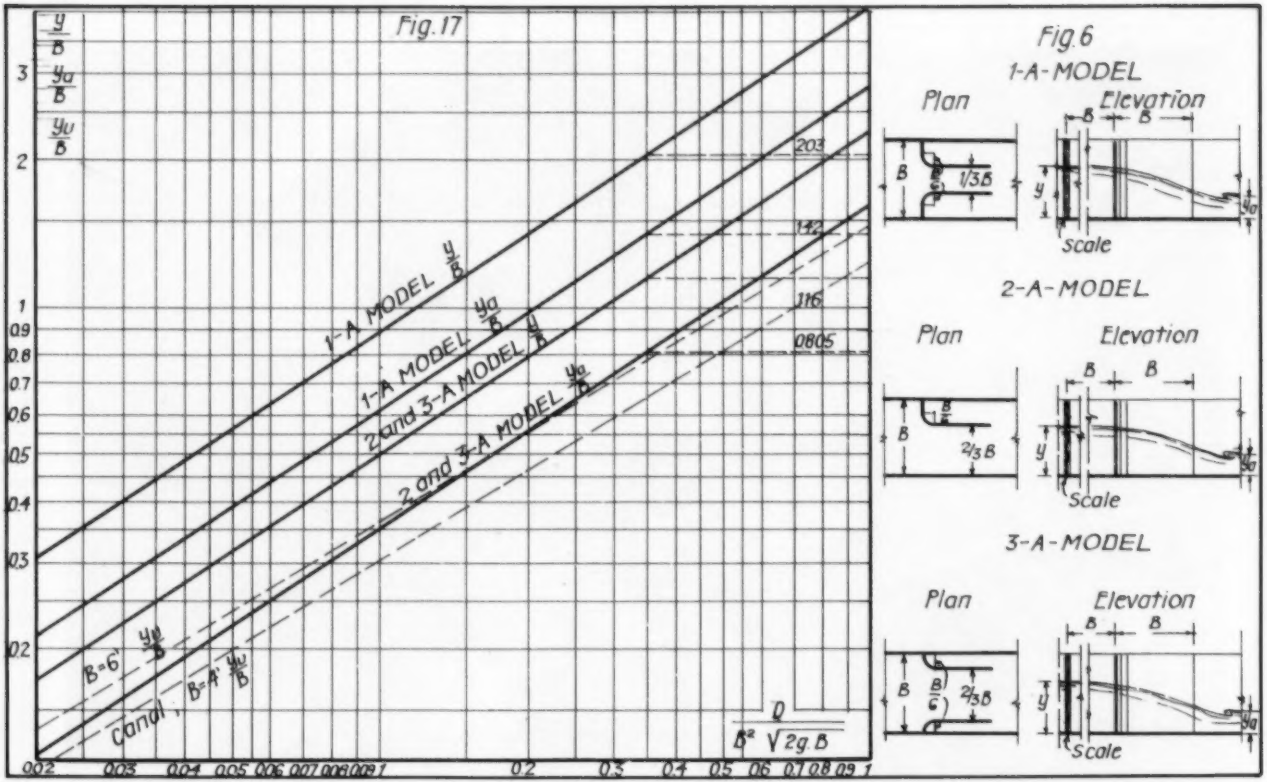


Fig.17
y/B
ya/B
yu/B
3
2
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
203
142
116
0805
1-A MODEL y/B
1-A MODEL ya/B
2 and 3-A MODEL y/B
2 and 3-A MODEL ya/B
B=6' yu/B
B=4' yu/B
Canal
Q / B² √2g B
0.02 0.03 0.04 0.05 0.06 0.07 0.08 0.09 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
Fig.6
1-A-MODEL
Plan
Elevation
B
1/3B
y
ya
Scale
2-A-MODEL
Plan
Elevation
2/3 B
Scale
3-A-MODEL
Plan
Elevation
2/3B
Scale

Fig. 7

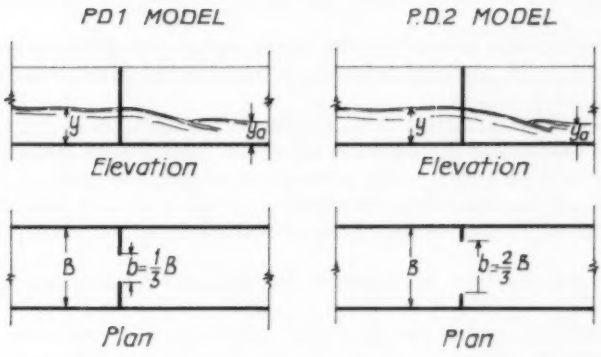
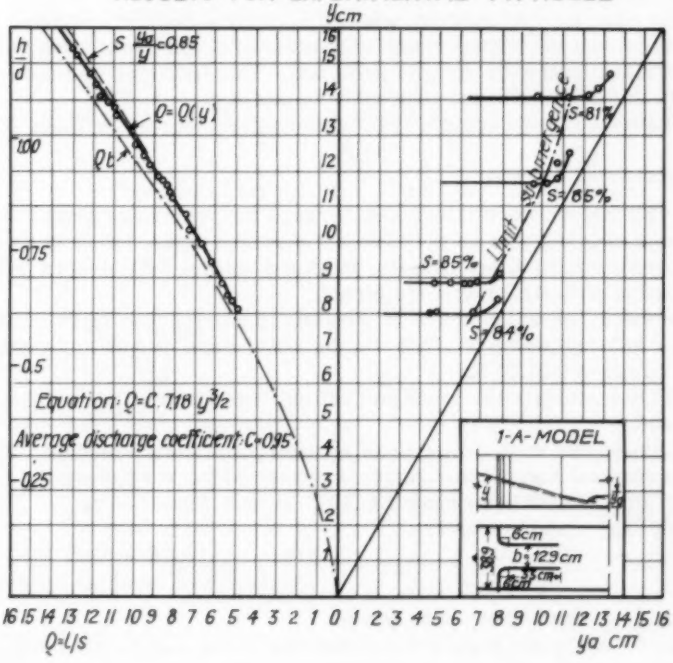


Fig. 8

RESULTS FOR EXPERIMENTAL 1-A MODEL



Experimental Results for "A" and "B" Models

Experimental results for "A" and "B" models are reported in Table I.

Experimental Results for "PD" Models

The "PD" models were intended to serve in temporary gaging stations as it is believed that they are the simplest means for measuring the discharge of a canal.

The throat width was assumed to be the distance between the edges of the two sheets. Similarly, the contraction was taken as the ratio of the gap to the width of the canal.

This procedure is analogous to that followed for orifices in closed conduits. The discharge coefficient includes the effect not only of head losses, but also the difference between throat width and section of critical flow.

In fact, it could be difficult to determine a critical section in a jet, so its depth was obtained by application of throat width and only for comparison purposes.

In the free-jet condition, that section was obtained downstream from the throat.

Since the critical section can vary with discharge, a variation in discharge coefficient might be expected. This is the first important difference between the "A" and the "PD" models.

The second effect of the location of the critical section is that the discharge coefficient cannot be safely applied to a meter without a level bottom. This was not the case for "A" models, where the critical regime remained in a known section and the bottom could be depressed downstream from the meter without influencing the discharge coefficient. This behavior was observed by De Marchi and Contessini for their models numbers 4, 5, 6, 7.

Experimental results for the "PD" meters are reported in Table II.

Standardization of Meters

Since "A" meters were intended to be used for permanent measuring stations, it was necessary to establish some standard types, for easy design or precast construction. Consequently, as stated before, only meters with 1/3 and 2/3 ratios of contraction were tested.

As a result of experiments, it is possible to adopt a discharge coefficient of .95 for r - 1/3 and .96 for r = 2/3. In the latter case, it does not matter if the meter is symmetrical or not.

As for the materials for walls and bottom, smooth concrete or sheet metal may be used.

Fig. 6 shows the main proportions adopted for standard models. In Fig. 17, a non-dimensional logarithmic graph is presented for easy design of meters. This diagram is based upon equation (12). In fact, since b = rB, $Q = C\mu rBy\sqrt{2gy}$, and therefore

$$\frac{Q}{B^2\sqrt{2gB}} = C\mu r(y/B)^{3/2} = \alpha(y/B)^{3/2}, \qquad (28)$$

in which α is a constant for each meter.

Citrini,[13] although using dimensional variables, has constructed some similar diagrams for the quoted De Marchi and Contessini models.

Generally, it is preferable to design the meter for a maximum permissible y_a, which corresponds to the maximum design value for S. It is believed that a safe value for S is .70. Then, equation (28) can be rewritten in terms of y_a:

Fig.9

RESULTS FOR EXPERIMENTAL 1-B-MODEL

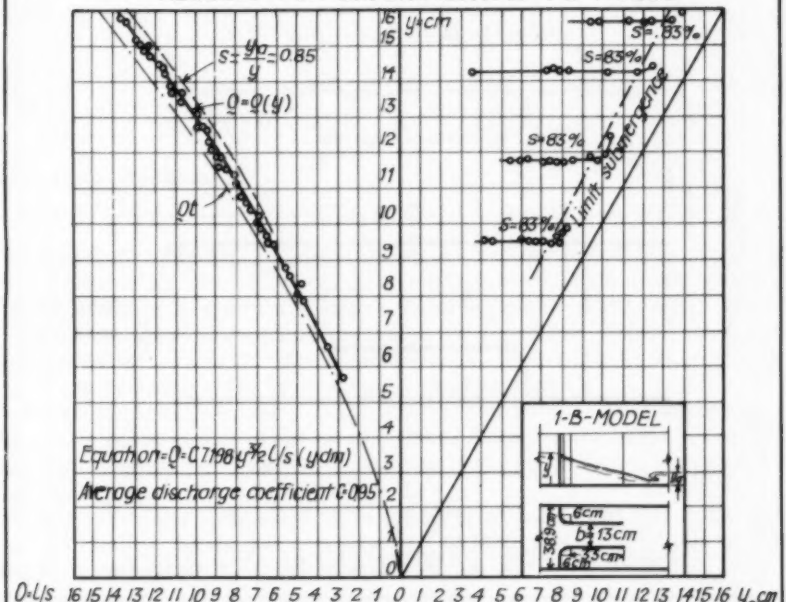


Fig.10

RESULTS FOR EXPERIMENTAL 2-A-MODEL

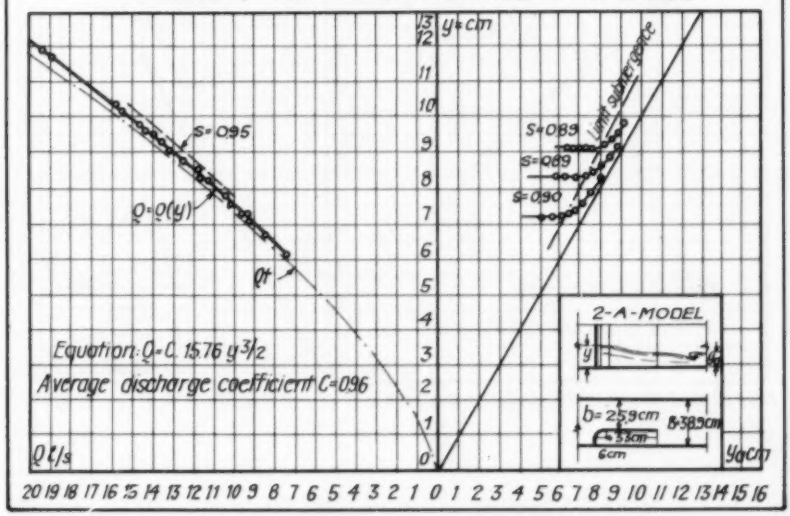


Fig. 11

IDEAL DOUBLE METER

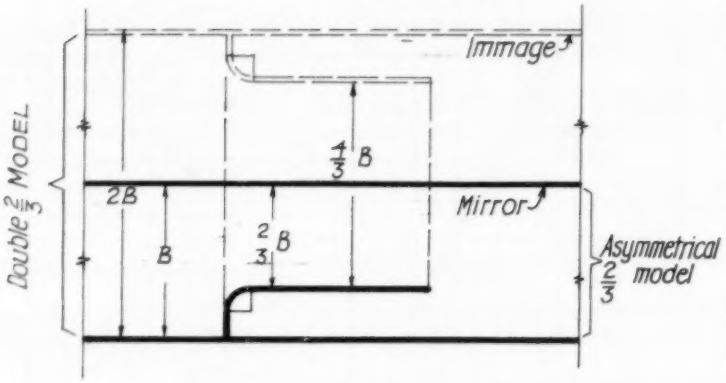
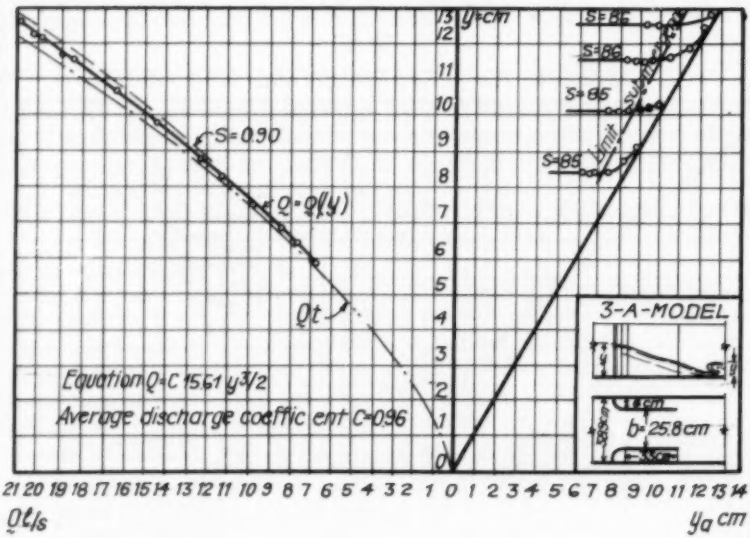


Fig 12

RESULTS FOR EXPERIMENTAL 3-A-MODEL



$$\frac{Q}{B^2\sqrt{2gB}} = \frac{C\mu r}{0.7^{3/2}} (y_a/B)^{3/2} = \alpha' (y_a/B)^{3/2} \quad , \tag{29}$$

in which α' is another constant, similar to α.

Equations (28) and (29) are plotted in Fig. 17 based on the following constants:

Model	r	C	μ	S_{max}
1A	1/3	.95	.397	.70
2A and 3A	2/3	.96	.434	.70

Examples of Application

At least two problems are involved in the design of a meter.

a) In a first instance, a meter may be desired for an existing canal. Hence, for any discharge, the meter will have a fixed downstream depth, which is the uniform depth or the corresponding undisturbed depth of the canal. In addition, the meter should not be submerged under any condition. On the other hand, since the meter is a disturbance for upstream canal regime, the corresponding backwater curve must lie within the existing banks.

For a first trial, assume a value of B for the meter. This width may or may not be equal to that of the canal. Then, calculate the values of $Q/B^2\sqrt{2gB}$ and corresponding y_u/B values, according to the y_u versus Q curve of the canal, where y_u is the undisturbed depth.

Two pairs of corresponding values determine a straight line in the graph of Fig. 17, which is the curve for the canal. In fact, referring mainly to uniform flow, the corresponding Chézy equation is:

$$Q = A\, c\, \sqrt{Rs} = K\sqrt{s} \quad . \tag{30}$$

in which:

A = Area of Flow;
c = Chézy coefficient;
R = Hydraulic radius;
s = Slope of canal;
K = Factor of conduction, after Bakhmeteff,[2] which is a function of y_u.

Also Bakhmeteff (op. cit.) observed that within usual requirements of precision:

$$K^2 = \text{Const. } y_u^n \quad , \tag{31}$$

in which n is a constant called the hydraulic exponent. Therefore:

$$Q = \beta y_u^{n/2} \quad , \tag{32}$$

in which β is another constant. Finally:

$$\frac{Q}{B^2\sqrt{2gB}} = \frac{\beta}{\sqrt{2g}\ \frac{5-n}{B^2}} \cdot (y_u/B)^{n/2} \quad ;$$

or

$$\frac{Q}{B^2\sqrt{2gB}} = \gamma (y_u/B)^{n/2} \quad . \tag{33}$$

Fig 13

RESULTS FOR EXPERIMENTAL P.D.1 MODEL

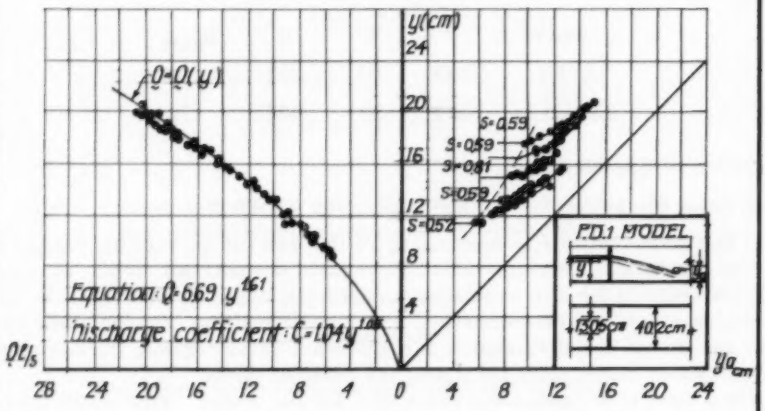


Fig. 15

RESULTS FOR EXPERIMENTAL P.D.2 MODEL

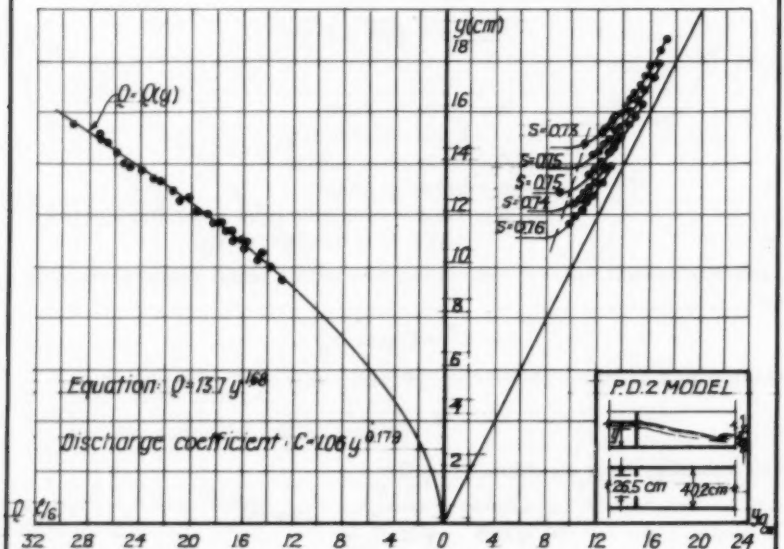


Fig. 14

COMPARISON BETWEEN PRESSURE HEADS AND SURFACE PROFILES FOR P.D.1 EXPERIMENTAL MODEL

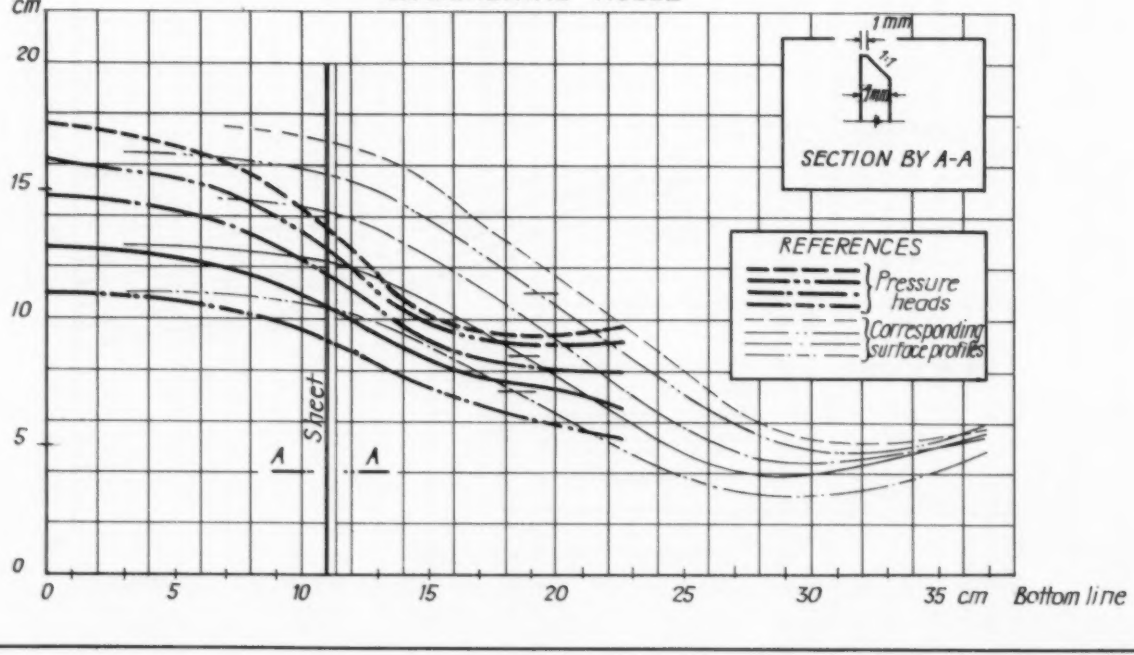
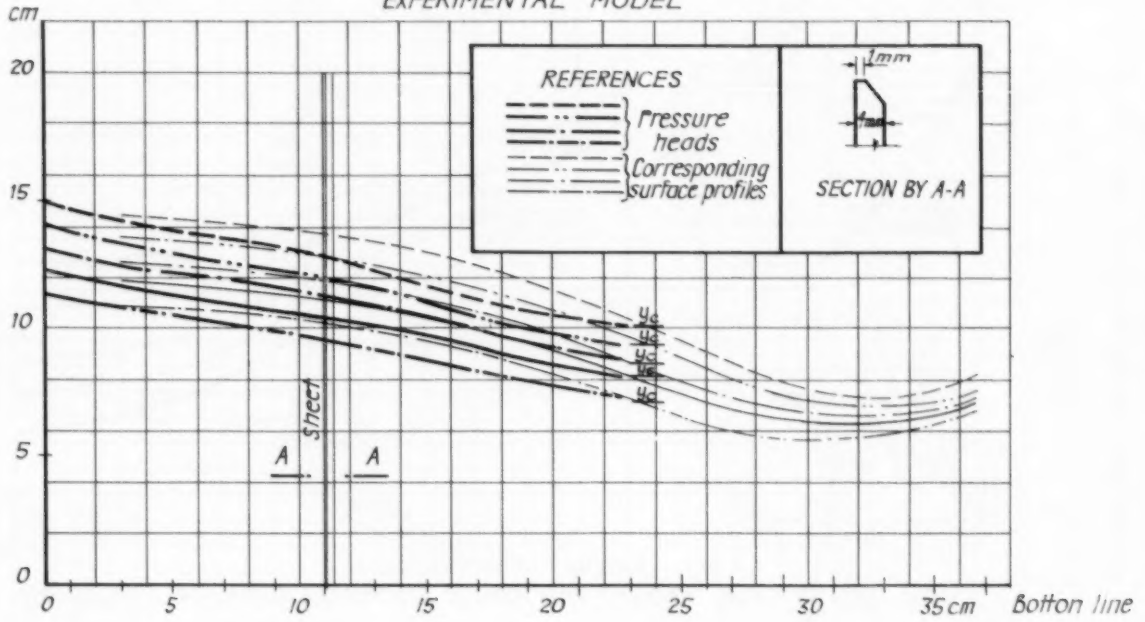


Fig 16

COMPARISON BETWEEN PRESSURE HEADS AND SURFACE PROFILES FOR P.D.2 EXPERIMENTAL MODEL



in which γ is another factor. Equation (33) can be represented in Fig. 17 as a straight line.

Thus, in the present problem, and in order that S never exceeds .70, this straight line must lie below the y_a/B line corresponding to the meter selected.

On the other hand, to prevent the water from overflowing the upstream banks, the y/B curve for the meter must always be below the values corresponding to maximum permissible depths.

If those two conditions cannot be obtained with the assumed value for B, another trial is made.

Once the dimensions of the meter have been obtained, by means of equation (12), the curve of discharge versus upstream depth is calculated more accurately. With this curve a scale is easily determined, in which each division shows the number of lit/sec or cfs that are passing through the meter at a given level. The scale must be located at the position shown by Fig. 6.

As an example, let an existing trapezoidal canal be assumed. Its bottom width is 6 ft and the bank slopes are 1:0.5 . The bottom grade is s = .001 . The walls and bottom have a roughness corresponding to a Manning's n of .014.

Alternatively, the meter bottom width will be assumed equal to 4 and to 6 ft. Thus with the aid of Woodward and Posey's tables,[15] the following conditions for uniform flow in the canal are obtained:

Q	y_u	B = 4 ft		B = 6 ft	
cfs	ft	$Q^2/B^2\sqrt{2gB}$	y_u/B	$Q^2/B^2\sqrt{2gB}$	y_u/B
56.6	2.0	.22	.50	-	-
181	4.0	.70	1.00	.26	.67
261	5.0	-	-	.37	.83

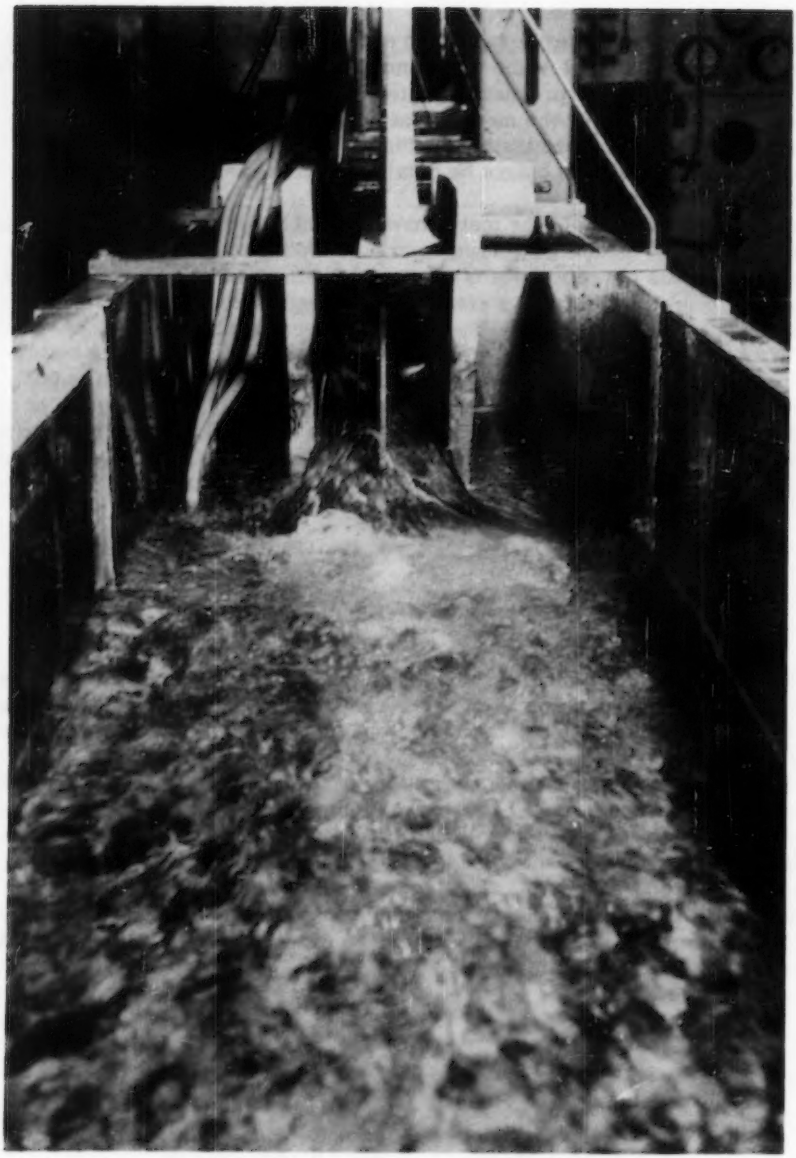
It is observed that when the meter width is B = 4 ft, the canal line lies below the y_a/B lines corresponding to all models (Fig. 17). Of course, for the 1A model, a greater distance occurs between the y/B and the y_u/B lines, i.e., there is a greater backwater than for the 2A or 3A models.

When B = 6 ft, the canal line cuts the y_a/B lines for the 2A and 3A models. This means that for the lower discharges the submergence for these models would be greater than .70. Theoretically a value of .80 could be reached and still have the meter correctly used. In spite of this, it is recommended not to exceed .70, until special conditions make it unavoidable.

If B = 4 ft is adopted, and the maximum design discharge is Q = 85 cfs, then for this maximum: $Q^2/B^2\sqrt{2gB} = 0.35$. With the aid of the graphs of Fig. 17, the following values are obtained:

Uniform depth of canal: $y_u = 0.66\ B = 2.6$ ft;
1A meter: $y = 2.03\ B = 8.1$ ft;
1A meter: $S = 2.6/8.1 = 0.32$;
1A meter maximum upstream elevation with respect to uniform flow: 8.1 - 2.6 = 5.5 ft;
2 or 3A meters: $y = 1.16\ B = 4.6$ ft;
2 or 3A meters: $S = 2.6/4.6 = 0.56$;
2 or 3A meters maximum upstream elevation with respect to uniform flow: 4.6 - 2.6 = 2 ft.

15. "Hydraulics of Steady Flow in Open Channels," Wiley, N. Y., 1941, p. 8.

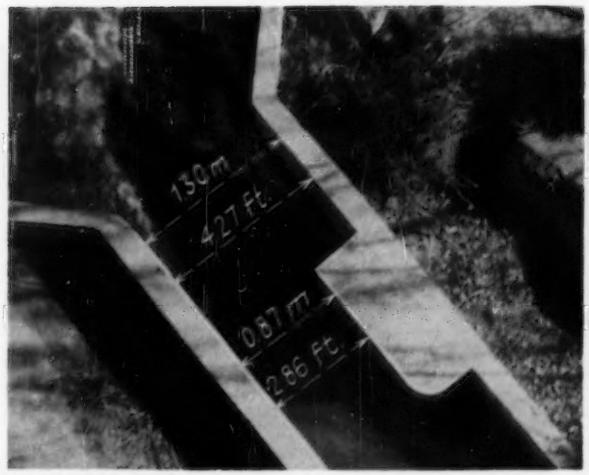


Photograph II.

Model 1 - A

Photograph III.

Prefabricated 2 - A Model

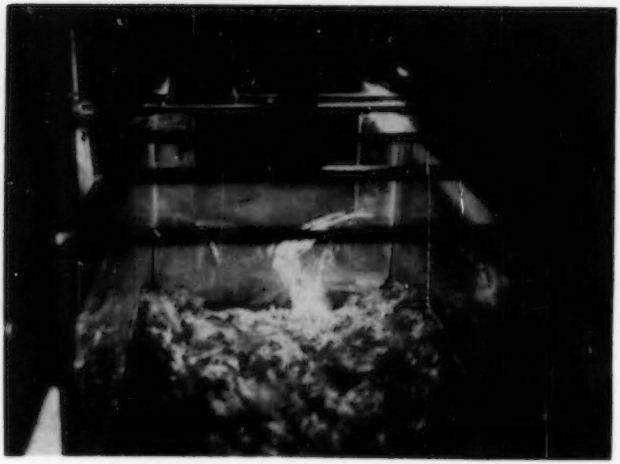


Photograph IV.

Poured Concrete 2 - A Model

Photograph V.

Emergency P D 1 Model



Photograph VI.

Experimental P D 2 Model

743-24

It is immediately noticed that the backwater caused by the 1A meter is greater than the usual freeboard. Therefore 2A or 3A meter would be more suitable.

If the water is silt laden, it would be necessary to keep the upstream velocity above the deposition value. In this case it would be perhaps desirable to have a bottom width of B = 5 ft.

b) A second case arises when the meter is to be built along with the canal. Then it is possible to design both canal and meter in such a way that upstream flow be either uniform, or accelerated or retarded. If the first of these conditions is imposed, the y_u/B curve must be identical to the y/B curve corresponding to the meter selected. Then the hydraulic exponent must be equal to 3, after equations (12) and (32). To fulfill the second condition (i.e. accelerated flow) the y_u/B curve must lie above the y/B curve, and vice versa for the third condition.

Thus the meter can be used not only for discharge measurements, but also to regulate the upstream level, since those depend exclusively on the values of the discharge.

Of course, the condition for maximum submergence must also be fulfilled in this case. This can be accomplished, for example, by means of a step, or by varying the bottom slope or the canal section downstream from the meter. This will be unnecessary if the meter produces a retarded backwater curve, as would be the case when it is placed in an existing canal.

Field Models

After the experiments were finished, several full-scale models were installed in the field.

One 2A model was used for measuring discharges up to 5 m^3/sec, (aprox. 175 cfs) and gave good results when checked with a current meter.

Another 2A model (Photograph III) was built of sheet metal and set in operation in an existing farm ditch in about 2 hours by untrained personnel. Its discharge range extended up to 100 lit/sec (approx. 3 cfs).

In Photograph IV is observed a 2A model of concrete, with range of discharges extending up to 2 m^3/ sec (approx. 70 cfs). For this model, a current meter check showed an error of 1.3% in a discharge of 750 lit/sec (approx. 28 cfs), as indicated by the scale of the critical flow meter. This error is well within the experimental error of a current meter operation.

In laboratory experiments for this model, the maximum discharge was 19.3 lit/sec (approx. .68 cfs), with y/B = .30. Conversely, for the model in Photograph IV, y/B = .48 and the discharge was 39 times the maximum laboratory discharge. This confirms the earlier statement about the safe extrapolation of laboratory results to greater dimensions and discharges.

Photograph V shows a field emergency PD1 model. Its sides were built with precast concrete plates, and the sheets were made with two iron plates welded to a frame.

Two other precast PD1 models were used in an existing canal in order to measure losses of discharge produced by infiltration in banks and bottom.

CONCLUSIONS

The models discussed herein are a simple and inexpensive means for measuring the discharge of a canal.

A safe extrapolation of laboratory results appears to be possible for larger

models as indicated by theoretical and experimental confirmations.

Models of type "A" can be precast and put in operation in little time.

Models of type "PD" require more experimentation in order to clarify completely the submergence limit and to obtain results for downstream depressed bottoms or for non-rectangular canals. This latter condition is very pertinent in the measure of discharge of existing canals in a practical manner without much preparation.

ACKNOWLEDGMENTS

The "A" models were studied in fulfillment of a scholarship granted by the Argentine Culture Board.

Prof. Rodolfo E. Ballester gave hearty encouragement to the work, which was accomplished with the aid of Mr. S. Notrica.

Prof. Francisco J. Dominguez, of the University of Chile, aided with his letter about the necessity of studying simple meters for open channels.

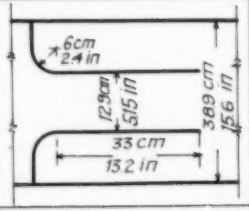
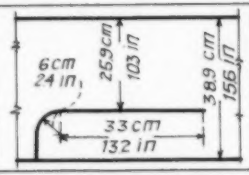
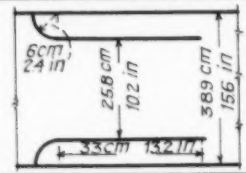
Prof. Giulio De Marchi, Director of the Istituto di Idraulica of the Politecnico di Milano, had the kindness to encourage this work.

Messrs. O. Lehmann and A. Velasco made all the experiments for the PD models and the pertinent computations.

The construction of models and the mechanical operation of the laboratory was accomplished by Mr. J. Cacciolo.

The author is very much indebted to Prof. Chilton Wright, M. ASCE, for going over the presentation of this paper, arrangement of the wording, and for many useful suggestions and encouragement.

TABLE I

Model	Sketch plan	$r = \frac{\text{Channel}}{\text{throat}}$	μ	Q_t	Material		Number of observations	Coefficient of Discharge		Q actual	Submergence limit S_1	Results plotted on fig
					bottom	throat walls		Mean value	Deviation from Mean			
				cfs					percent	cfs		
1A		0.332	0.397	$3.18 by^{3/2}$	Concrete	Concrete	64	0.948	0.62	$3.01 by^{3/2}$	0.80	8
1B	Same as for 1A	0.334	0.39	$3.20 by^{3/2}$	Concrete (smoothed)	lucite	42	0.95	0.42	$3.04 by^{3/2}$	0.83	9
2A		0.666	0.435	$3.48 by^{3/2}$	Concrete	Concrete	19	0.962	0.5	$3.34 by^{3/2}$	0.90	10
3A		0.663	0.432	$3.46 by^{3/2}$	Concrete	Concrete	37	0.956	0.7	$3.32 by^{3/2}$	0.85	12

REMARKS - Table I

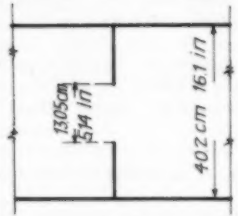
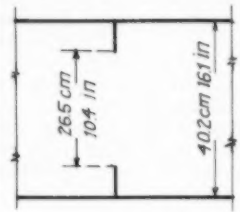
1A Variations of C are accidental within a 1% error. Free-flow results on left side of fig. 8 compared with theoretical free-flow curve. Four different discharge values used for point-of-submergence determinations. Results on right side of fig. 8 show variations of Y with respect to Y_a. For free flow, each curve is parallel to the Y_a axis. For submerged flow the curves slope upward. De Marchi and Contessini's 12 and 13 models showed similar submergence characteristics with C = 0.98. Parshall flume has $S_1 \angle 0.70$. For 1-A model S_1 only 10% lower than theoretical value. On left side of fig. 8 is plotted the Q vs Y curve for S = 0.85, with submerged flow but still moderate error.
Free-surface profiles and corresponding bottom pressures on center line, for several discharges, show raising of pressure curve for upward concave surface and vice-versa. However, the differences between both series of curves are small, because of small curvature of flow in the throat. Two oblique waves formed on water surface, springing from entrance circular curves, crossing each other near critical section. Critical section always within the throat. Photograph II shows from downstream this experimental model with free-jet condition.

1B Used for observation of effect of wall and bottom roughness. Remarks as for 1-A model. There is no difference of results within the order of magnitude of experimental errors.

2A Asymmetrical model. Discharge coefficient higher than for 1A and 1B models. Flow pattern along the sheet quite similar to the 1A model with standing oblique wave.
The asymmetry of this model is only apparent because the canal wall was very smooth, and its influence could be neglected. Thus, this model would behave quite similarly to one obtained by reflection on a mirror (fig. 11), located at the channel wall. Under this condition the ideal meter would have a double width of throat and upstream canal, i.e. the same contraction ratio for double discharge. This ideal meter would be symmetrical. Theoretical submergence limit was practically reached. De Marchi and Contessini's 4, 11, 15 models had comparable features. At low discharges 4 and 11 models gave greater submergence limits, but 2A model produced greater values for higher discharges.

3A Discharge coefficient very close to that of the 2A model, which confirms the above statements about its asymmetry and is a first confirmation of the possibility of safe extrapolation of results to larger dimensions. De Marchi and Contessini's comparable 4, 11 and 15 models had C coefficients between 0.96 and 0.978. This may substantiate the assumption that gradual expansion of the exit would be useless for diminishing the head loss. De Marchi and Contessini's 4, 5, and 11 models had lower submergence limits for comparable discharges.

TABLE II

Model	Sketch plan	Channel throat	μ	Qt	Material		Number of Observations	Coefficient of Discharge			Q actual		Submergence limit S1	Results plotted on fig.
					bottom	throat walls		Mean Value	Deviation from Mean	$C = Ay^n$	Mean C	$C = Ay^n$		
				cfs										
PD1		0.325	0.396	$3.17 by^{3/2}$	Concrete	lucite sheets	111	0.98	1.2	$1.04y^{0.105}$	$3.11 by^{3/2}$	$3.30 by^{1.67}$	0.52	13
PD2		0.659	0.426	$3.42 by^{3/2}$	Concrete	lucite sheets	32	0.92	2.2	$1.06y^{0.179}$	$3.15 by^{3/2}$	$3.63 by^{1.68}$	0.75	15

REMARKS - Table II

PD 1. Mean value of C = 0.98. Mean deviation of 1.2% not accidental. In a logarithmic plot of values Y and C, a straight line was fitted by means of least squares. Correlation coefficient was 0.91 and C = 0.925 Y 0.105 (Y in decimetres) C = $1.04Y^{0.105}$ (Y in ft). More scattering of submergence runs when compared with A and B models. Submergence limit always lower than for A and B models.
Determinations made with the series of point gages and the manifold manometer shown in fig. 14. Critical depth location referred to throat width did not vary very much. Profiles made along centerline. Curvature upward convex corresponded to lowering of pressure lines and vice-versa, in a very definite trend.

PD 2 Correlation between log C and log Y = 0.83, lower than that for PD 1 model. C = 0.868 Y^{0179} (Y, decimetres) C = 1.06 $Y^{0.179}$ (Y, ft)
Rather difficult to establish definite submergence limits. Minimum values of S corresponded to the free jet tests. All these coefficients were close to 0.75 and greater than S_1 for PD 1 model. Since the bottom was level it was impossible to obtain even lower values of S. The values reported herein for S_1 must be taken as provisional. Profiles and diagrams of bottom pressures are plotted on fig. 16. They show less curvature than for the PD 1 model.
Critical depth was rather stable.
Photograph VI shows the experimental PD 2 model.

PROCEEDINGS PAPERS

The technical papers published in the past year are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

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JULY: 457(AT), 458(AT), 459(AT)[c], 460(IR), 461(IR), 462(IR), 463(IR)[c], 464(PO), 465(PO)[c].

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)[c], 479(HY)[c], 480(ST)[c], 481(SA)[c], 482(HY), 483(HY).

SEPTEMBER: 484(ST), 485(ST), 486(ST), 487(CP)[c], 488(ST)[c], 489(HY), 490(HY), 491(HY)[c], 492(SA), 493(SA), 494(SA), 495(SA), 496(SA), 497(SA), 498(SA), 499(HW), 500(HW), 501(HW)[c], 502(WW), 503(WW), 504(WW)[c], 505(CO), 506(CO)[c], 507(CP), 508(CP), 509(CP), 510(CP), 511(CP).

OCTOBER: 512(SM), 513(SM), 514(SM), 515(SM), 516(SM), 517(PO), 518(SM)[c], 519(IR), 520(IR), 521(IR), 522(IR)[c], 523(AT)[c], 524(SU), 525(SU)[c], 526(EM), 527(EM), 528(EM), 529(EM), 530(EM)[c], 531(EM), 532(EM)[c], 533(PO).

NOVEMBER: 534(HY), 535(HY), 536(HY), 537(HY), 538(HY)[c], 539(ST), 540(ST), 541(ST), 542(ST), 543(ST), 544(ST), 545(SA), 546(SA), 547(SA), 548(SM), 549(SM), 550(SM), 551(SM), 552(SA), 553(SM)[c], 554(SA), 555(SA), 556(SA), 557(SA).

DECEMBER: 558(ST), 559(ST), 560(ST), 561(ST), 562(ST), 563(ST)[c], 564(HY), 565(HY), 566(HY), 567(HY), 568(HY)[c], 569(SM), 570(SM), 571(SM), 572(SM)[c], 573(SM)[c], 574(SU), 575(SU), 576(SU), 577(SU), 578(HY), 579(ST), 580(SU), 581(SU), 582(Index).

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c. Discussion of several papers, grouped by Divisions.

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